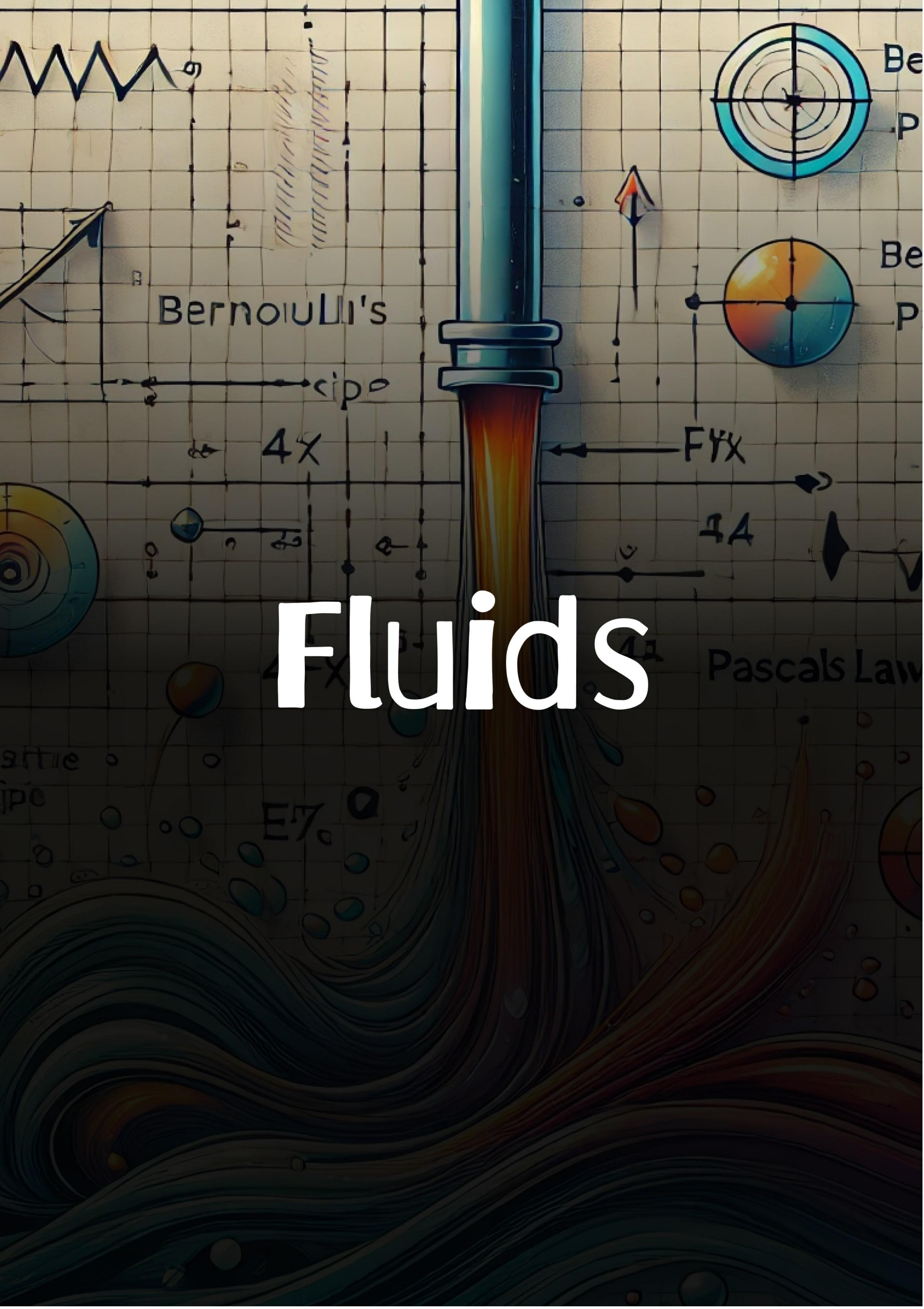


Fluids

fluids

PRESSURE

At any instant molecules of a fluid are in random motion in all directions. Let there be a small chit of area dA immersed in a fluid and let dF be the force experienced by it on one phase due to collisions with the fluid particles. Then we define pressure at this point as

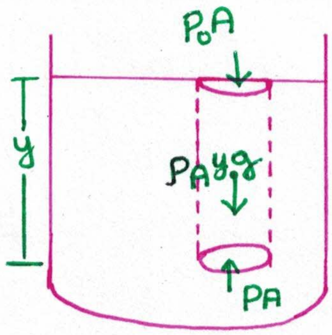
$$P = \frac{dF}{dA}$$

NOTE: Pressure always acts perpendicular to a surface and thus total pressure force on any surface can be found as

$$F = \int P\,dA$$

$$\boxed{F = \int P.dA}$$

VARIATION OF PRESSURE WITH DEPTH IN A VESSEL



(P_0 = atm pressure)

$$P_0A = \rho A y g = PA$$

$$P = P_0 + \underbrace{\rho y g}_{\text{guage pressure}}$$

In a non-accelerated vessel, pressure at same depth is same.

BUOYANT FORCE

The resultant of all the pressure forces acting on a submerged object is called buoyant force.

Statement: The buoyant force acting on any immersed object is equal to the weight of the displaced volume of liquid.

Displaced liquid means volume of the submerged object below the free surface.

GUAGE PRESSURE

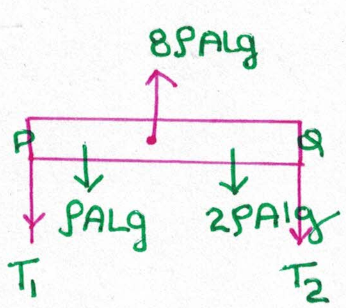
The difference b/w actual pressure and atmospheric pressure is called guage pressure.

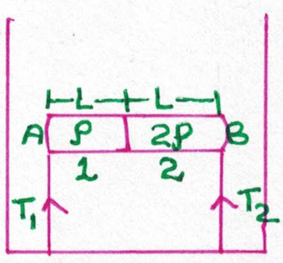
NOTE: For most of the problems we can find the resultant pressure force just by integrating the guage pressure because the P_0 term cancel out.

CENTRE OF BUOYANCY

The point at which the resultant buoyant force acts is called centre of buoyancy. This point is same as the centre of mass of the displaced liquid.

Que.) 1 and 2 are ~~two~~ different cylinder which is in equilibrium





$$\tau_P = 0$$

$$T_2 \times 2L + 2\rho ALg \times \frac{3L}{2} + \rho ALg \times \frac{L}{2} = 8\rho ALg \times L$$

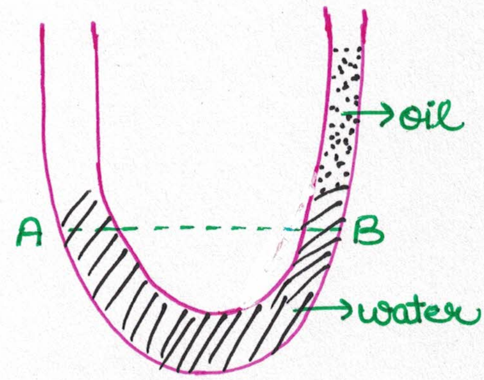
$$T_2 = \frac{9\rho AL^2 g}{2 \times 2\cancel{L}} = \frac{9}{4}\rho ALg$$

$$T_1 = 8\rho ALg - 3\rho ALg - T_2$$

$$T_1 = \frac{11}{4}\rho ALg$$

NOTE: In a continuous column of stationary liquid, pressure at same depth is same.

Que.) Compare the pressure at A & B.



$P_B > P_A$

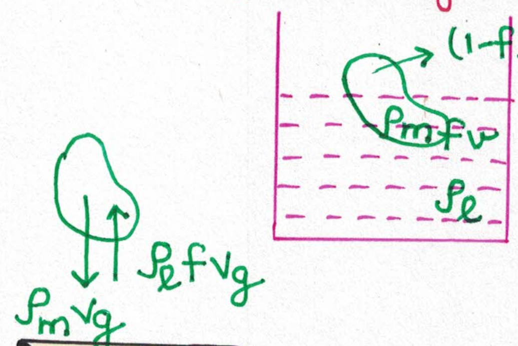
as the fig. ($\rho y g$) will be subtracted from both sides & ρ of oil is less. Therefore

$P_B > P_A$

Que.) what is the fraction of the volume of submerged object.

$$\rho_m \cdot V g = \rho_\ell f V g$$

$$f = \frac{\rho_m}{\rho_\ell}$$

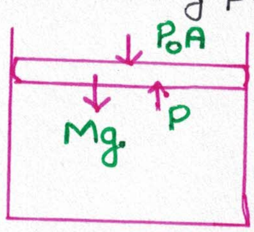


PASCAL'S LAW

Pressure in a confined liquid is increased at any point then it increases by the same amount at every point.

$$P_0 A + Mg = PA$$

$$P - P_0 = \frac{Mg}{A}$$



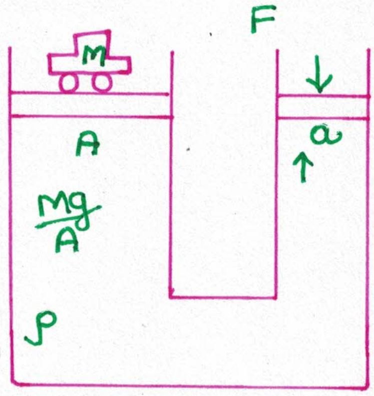
If pressure is increased by Mg/A, then Mg/A will be added at each point to $P_0 + \rho y g$.

HYDRAULIC LIFT

$$F = \frac{Mg}{A} a$$

If $\frac{a}{A} << 1$

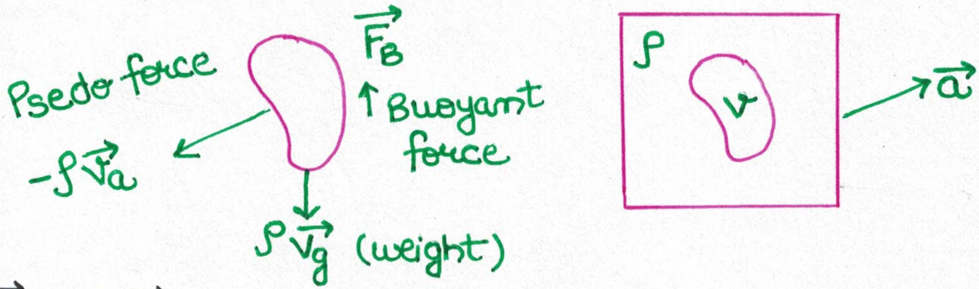
then $F << Mg$



BUOYANT FORCE IN ACCELERATED VESSELS

In accelerated vessel apart from the usual buoyant force an additional force acts in the direction of the acceleration of the vessel. Its magnitude is given by mass of the displaced liquid times the acceleration of the vessel.

It can be shown as follows:



$$\vec{F_B} + \rho\vec{V}\vec{g} - \rho\vec{V}\vec{a} = 0$$

$$\vec{F_B} = \underbrace{-\rho\vec{V}\vec{g}}_{\text{usual buoyant force}} + \rho\vec{V}\vec{a} \longrightarrow \text{additional buoyant force}$$

$$\boxed{\vec{F}_{B\,additional} = \rho\vec{V}\vec{a}}$$

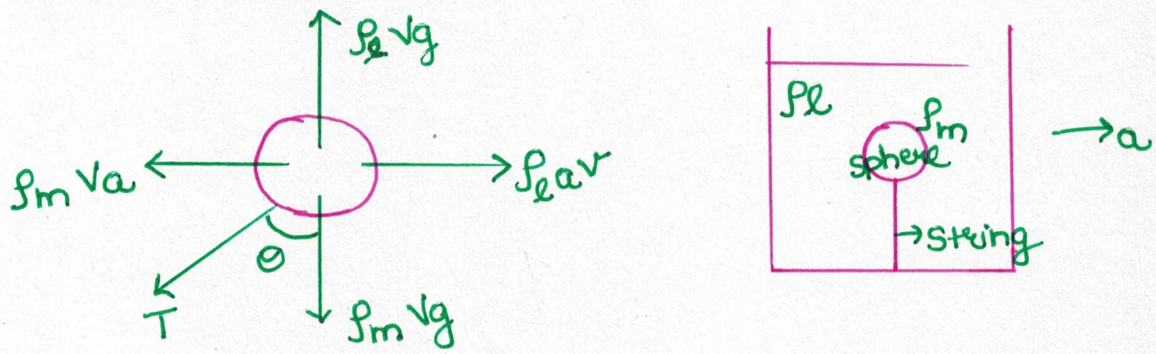
NOTE: If we like, we can define $\vec{g'} = \vec{g} - \vec{a}$ & say that, $\vec{F_B} = -\rho\vec{V}\vec{g'}$

Que.) A sphere is tied to the bottom of the vessel as shown in the fig. Now the vessel is accelerated:

(i) Draw FBD in the vessel frame

(ii) Calculate the inclination of the thread from vertical.

(iii) Calculate tension.

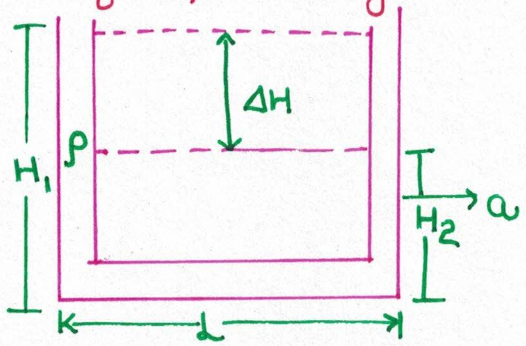


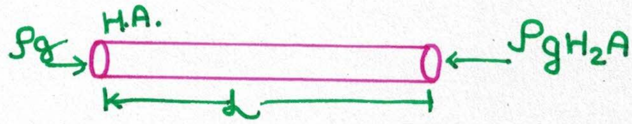
$$T\cos\theta + \rho_m Vg = \rho_l Vg$$

$$T\cos\theta = \rho_l Vg - \rho_m Vg$$

$$T\sin\theta = \rho_l aV - \rho_m Va \qquad \left(\tan\theta = \frac{a}{g}\right)$$

Que.) A bent tube is accelerated as shown. Find ΔH in terms of a, L and g.

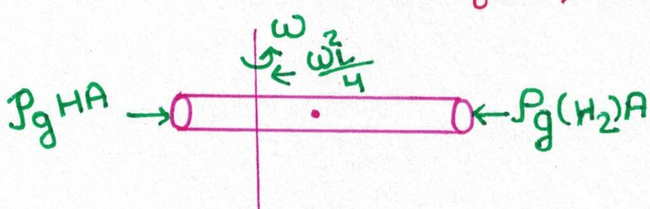


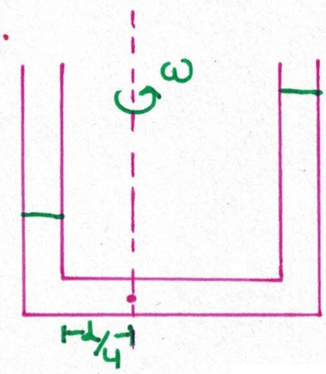


$$\rho A L a = \rho g (H_1 - H_2) A \qquad (ma = F_{net})$$

$$\Delta H = \frac{aL}{g}$$

Que.) Find ΔH in terms of ω, L & g.



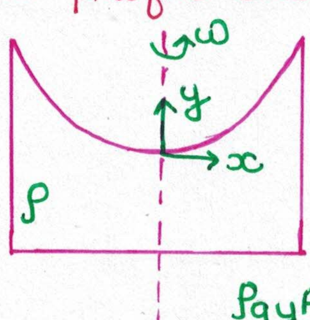


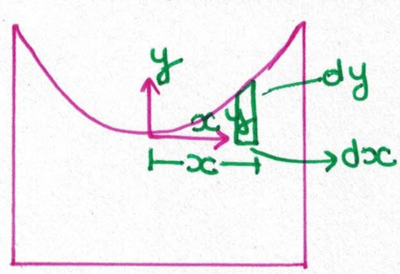
$$F_{net} = ma_{cm}$$

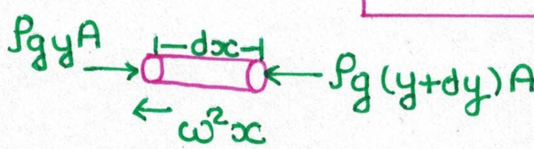
$$\rho g (H_2 - H_1) A = \rho A L \times \frac{\omega^2 L}{4}$$

$$\Delta H = \frac{\omega^2 L^2}{4g}$$

Que.) For the fluid rotating with ω as shown, find the eq^n of surface profile and verify that its a parabola.







$$\rho g (y + dy) A - \rho g y A = \rho A\, dx \times \omega^2 x$$

$$\int_0^y g \cdot dy = \int_0^x \omega^2 x \cdot dx$$

$$y = \left(\frac{\omega^2}{2g}\right) x^2$$

Ideal flow

A flow is said to be ideal if
(i) it is non-viscous
(ii) it is incompressible
(iii) steady
(iv) uniform
(v) stream lined
(vi) irrotational

NON - VISCOUS

A liquid is non-viscous if there is no stickness (friction) between its particles.

INCOMPRESSIBLE

Density does not change with change in pressure.

STEADY

If the velocity at any point in the duct does not change with time, it is said to be a steady flow.

UNIFORM

If the velocity of all the particles at a given cross section is same, we say flow is uniform.

STREAM LINES

Stream lines are drawn to visualize the fluid flow. Their density is proportional to the fluid velocity & the tangent at any point of streamlines gives the direction of particle velocity.

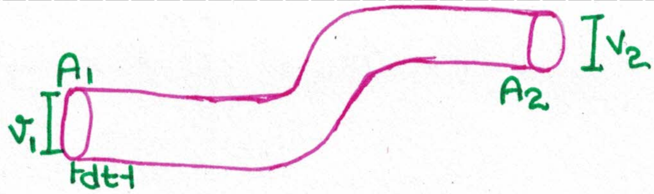
* opposite to streamline flow is turbolent flow.

IRROTATIONAL FLOW

A flow is irrotational if there is no component of velocity perpendicular to bulk motion of the substance.

CONTINUITY EQUATION

At any cross-section of a pipe carrying incompressible fluid, the product of cross-sectional area & velocity is constant.



(small volume) $d\forall = A_1 v_1 dt$ | $d\forall = A_2 v_2 dt$

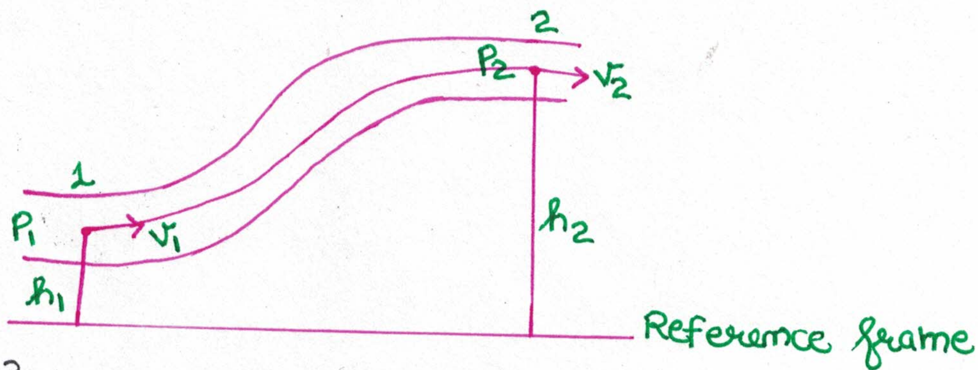
(rate flow) $\dfrac{d\forall}{dt} = Q = A_1 v_1$ | $\dfrac{d\forall}{dt} = A_2 v_2$

rate of inflow = rate of outflow

$$\boxed{Q = A_1 v_1 = A_2 v_2}$$

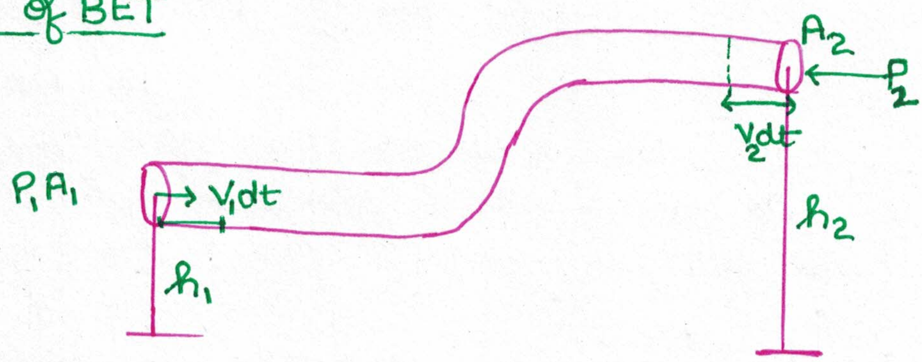
Q is volume flow rate.

BERNOULLI'S THEOREM (BET)



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Proof of BET



$$dw_p = P_1 A_1 v_1 dT - P_2 A_2 v_2 dt$$

$$dW_p = Q\,(P_1 - P_2)\,dt$$

$$dW_g = -dw = -(\rho Q dt)\,g\,(h_2 - h_1)$$

$$dW_g = \rho Q g\,(h_1 - h_2)\,dt$$

$$dK_E = \frac{1}{2}(\rho Q dt)\,(v_2^2 - v_1^2)$$

$$dW_p + dW_g = dKE$$

$$\cancel{Q dt}\left[(P_1 - P_2) + \rho g (h_1 - h_2)\right] = \cancel{Q dt}\left[\frac{1}{2}\rho\,(v_2^2 - v_1^2)\right]$$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2}$$

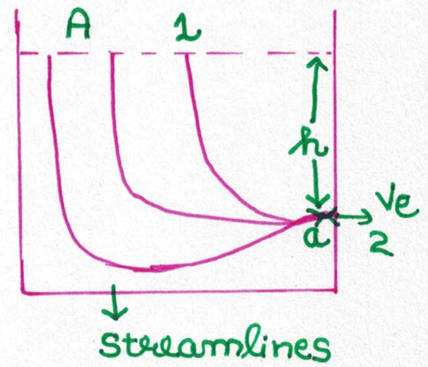
NOTE : If a liquid is falling freely unsurrounded by a duct, then the pressure in the liquid is same as atmospheric pressure.

TORRICELLI'S LAW (TOL)

In a vessel of wide cross-section, if there is a hole of small cross-section at a depth h below the free surface then the efflux velocity (v_e) is given by

$$v_e = \sqrt{2gh} \qquad (\text{when } a <<< A)$$

Proof:



using BET $(1 \to 2)$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

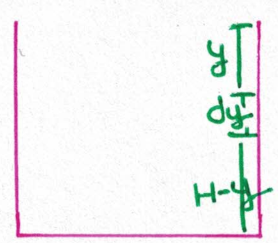
$$\begin{array}{cccccc} P_1 & v_1 & h_1 & P_2 & v_2 & h_2 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ P_0 & 0 & h & P_0 & v_e & 0 \end{array}$$

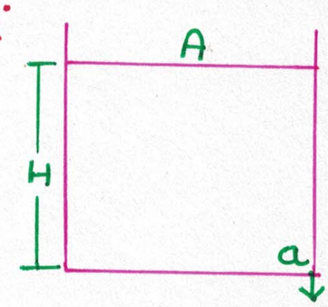
$$P_0 + \rho g h = P_0 + \frac{1}{2}\rho v_e^2$$

(v_1 can be taken as zero practically because $a <<< A$)

$$v_e = \sqrt{2gh}$$

Que.) A jug is filled to a height H as shown, After how much time will the level become $H/2$.





$$v_e = \sqrt{2g(H-y)}$$

$$Q = \sqrt{2g(H-y)}\,a$$

$$dV = \sqrt{2g(H-y)}\,a.dt = A.dy$$

$$\int_0^{H/2} \frac{dy}{\sqrt{2g(H-y)}} = \frac{a}{A}\int_0^{T_{1/2}} dt$$

Que.) Find $v_e = ?$ & $P_B = ?$

BET $(1 \to 2)$

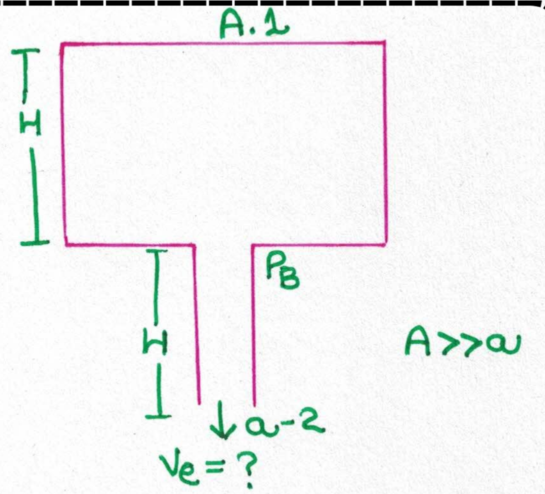
$$\cancel{P_0} + \cancel{\frac{1}{2}\rho(0)^2} + \rho g(2H) = \cancel{P_0} + \frac{1}{2}\rho v_e^2 + \cancel{\rho g(0)}$$

$$v_e = 2\sqrt{gH}$$

BET $(B \to 1)$

$$P_B + \rho g H = P_0$$

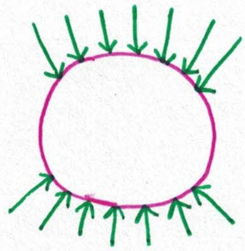
$$P_B = P_0 - \rho g H$$



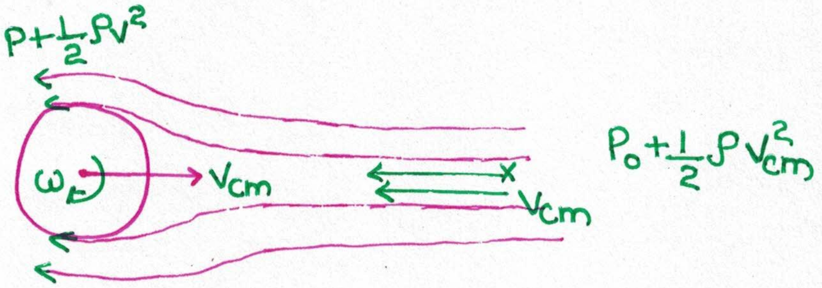
APPLICATIONS OF BERNOULLI'S THEOREM

1. MAGNUS EFFECT (SWINGING OF A CRICKET BALL)

Consider a ball moving to the right with speed v_{cm}, having an angular velocity ω as shown. In the frame, of center of the ball, the bulk of air has velocity to the left. Due to the spin of the ball, molecules of air at the top tend to slow down and at the bottom speed up. Due to bernoulli's theorem, $(P + \frac{1}{2}\rho v^2)$ is constant. Accordingly where velocity is high the pressure is low and vice-versa. The pressure forces acting on the ball are as shown.

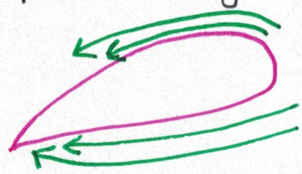


★ Therefore, the range of the ball will be less than expected.



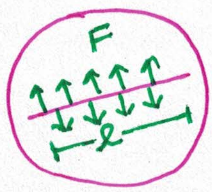
2. AERO FOIL

The cross section of an aero foil is such that below the wing, the velocity of air is smaller as compared to velocity at the top. Accordingly, there is high pressure below the wing and low pressure above. This results in a net lift on the plane wing.



SURFACE TENSION

The surface of any fluid behaves like a stretched membrane due to attractive cohesive forces between the surface molecules. This attractive force per unit length on the surface is called surface tension.

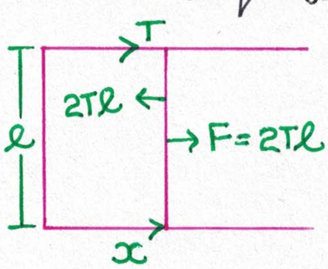


$$T = \frac{F}{l}$$

SURFACE ENERGY

Negative of work done by surface tension forces is called surface potential energy.

Whenever we make a soap film, we make two surfaces (surface means liq. - air interface)



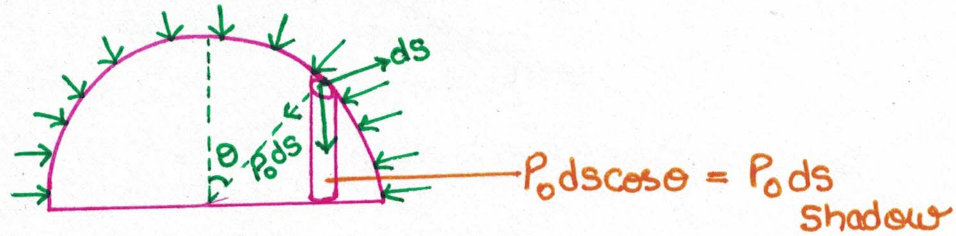
$$W_T = -2Tlx$$
$$U_T = 2Tlx$$
$$A = 2lx$$
$$\boxed{\frac{U_T}{A} = T}$$

*Relationship b/w surface tension and surface energy.

Surface energy per unit surface area is called surface tension.

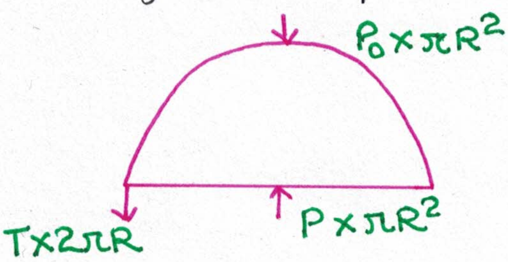
NOTE: If an area A is titled through an angle θ from the perpendicular configuration of a light beam, then its shadow area becomes A Cosθ on a plane perpendicular to the light.

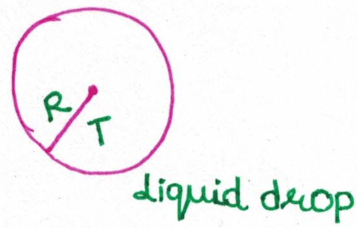
EXCESS PRESSURE INSIDE A LIQUID DROP



$\therefore$ Total pressure $= \int P_0 \, ds_{shadow} = P_0 \pi R^2$

(horizontal components will cancel out)





$$P_0 \times R^2 + 2\pi RT = P\pi R^2$$

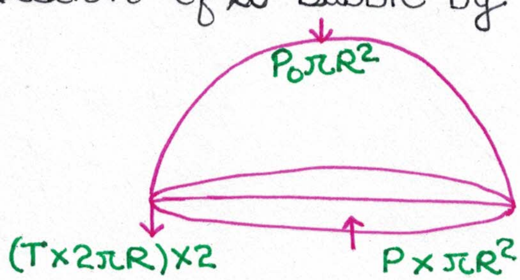
$$P - P_0 = \frac{2T}{R}$$

$$\boxed{\Delta P = \frac{2T}{R}}$$

Even though we have proved this for a sphere the same result holds even for a section of liquid sphere. The pressure on the concave side is higher than the convex side.

EXCESS PRESSURE IN A SOAP BUBBLE

The pressure on the concave side is higher than the convex side for section of a bubble by $\frac{4T}{R}$.



$$P_0 \pi R^2 + 4\pi RT = P\pi R^2$$

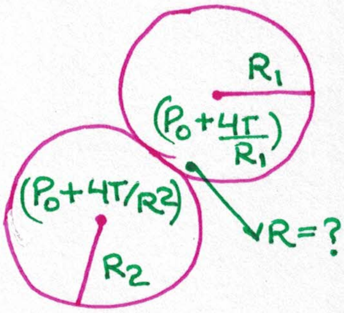
$$P - P_0 = \frac{4T}{R}$$

$$\boxed{\Delta P = \frac{4T}{R}}$$

Que.) Find the radius of curvature of the common film of soap bubble.

$$\left(P_0+\frac{4T}{R_1}\right)-\left(P_0+\frac{4T}{R_2}\right)=\frac{4T}{R}$$

$$R=\frac{R_1R_2}{R_2-R_1}$$



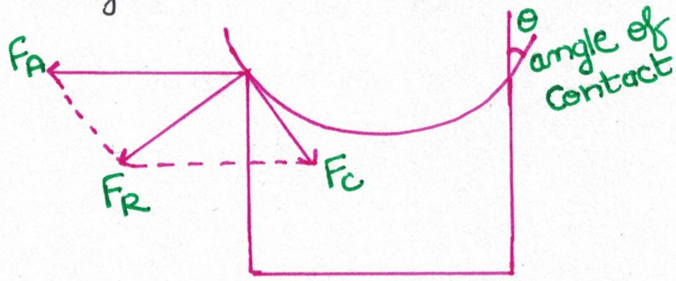
MENISCUS

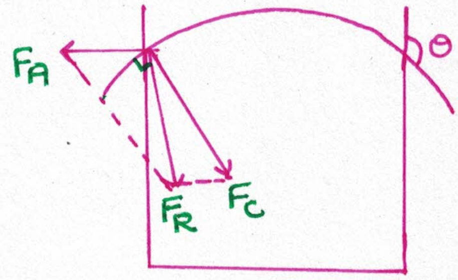
Consider any molecule near the material of the vessel. There are two forces acting on it:

(i) Cohesive forces due to attraction b/w the molecules of the liquid.

& (ii) Adhesive forces due to attraction b/w the molecules of the liquid & molecules of the material of the vessel.

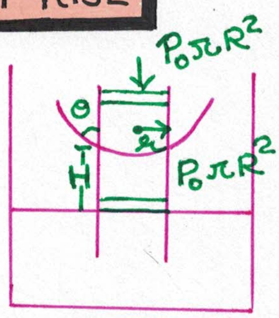
A liquid cannot take any shearing force in the steady state. Accordingly, the surface of liquid is perpendicular to the resultant of these forces. Depending on the resultant being inside or outside the vessel, two cases arise as shown:

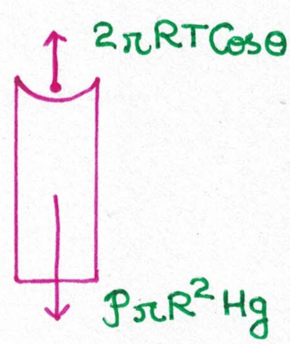




NOTE: Angle of contact is a property of a pair of materials.

CAPILLARY RISE



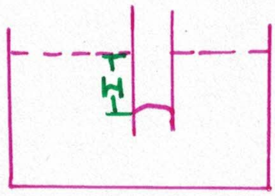


$$2\pi RT\cos\theta=\rho\pi R^2Hg$$

$$\boxed{H=\frac{2T\cos\theta}{\rho r g}}$$

CAPILLARY FALL

In the above formula, if θ is obtuse we get a negative value of H which means that the level in the capillary will be below.

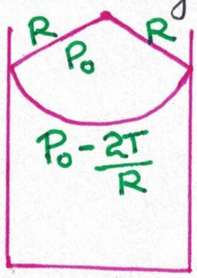


CAPILLARY OF INSUFFICIENT HEIGHT OR DEPTH

If the capillaries of insufficient length then liquid will rise all the way to bottom such that,

$$H = \frac{2T\cos\theta}{\rho r g}$$

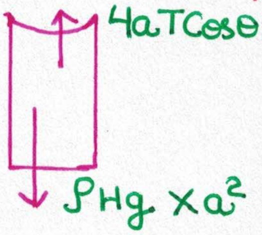
is still valid and angle of contact will adjust itself.



Que:) Derive an expression for capillary rise in a capillary of square section.

$$4aT\cos\theta = \rho Hga^2$$

$$H = \frac{4T\cos\theta}{\rho g a}$$



VISCOUSITY

It is related to stickness of a fluid (friction b/w the layers).

NEWTON'S LAW FOR VISCOUSITY

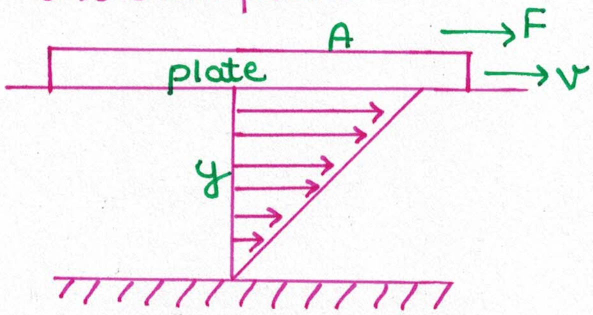
Consider a plate of area A being dragged on a pond of liquid with a velocity v as shown. Then the viscous drag force experienced by the plate is given by

$$F_v = -\eta A\frac{dv}{dy}$$

η is called coefficient of viscousity. Its S.I. unit is deca poise.

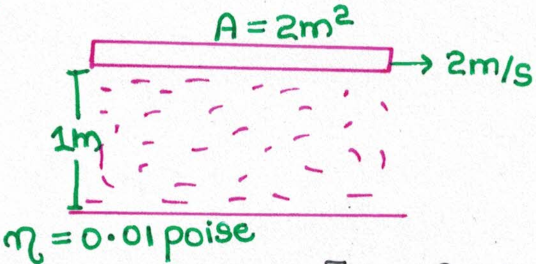
velocity gradient $= \frac{dv}{dy} = \frac{v}{y}$ (for newtonian fluids)

10 poise = 1 deca poise
1 poise = 0.1 deca poise



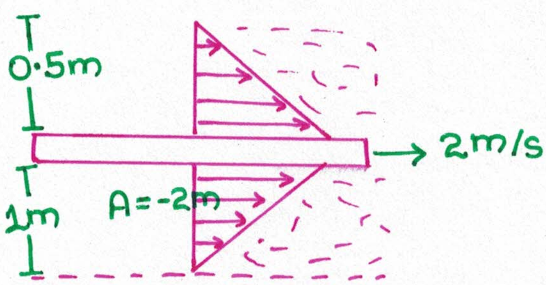
Force will be opposite to the motion of the liquid layers.

Que.)



$$F = -\frac{2\times1\times2}{1000} = -4\times10^{-3}N$$

Que.)



$\eta = 0.01$ poise

$$F = -(4\times10^{-3} + 8\times10^{-3})$$
$$= -12\times10^{-3}N$$

STOKE'S LAW

If a sphere of radius R is moving in an infinite pond with a speed v then the viscous drag acting on it is given by

$$F_v = -6\pi\eta Rv$$

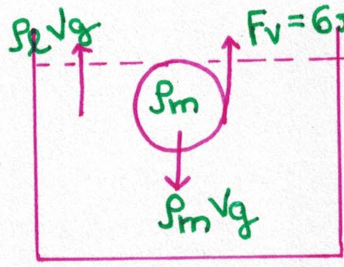
(opposite to the velocity of sphere)

TERMINAL VELOCITY

$$6\pi\eta R v + \rho_l V g = \rho_m V g$$

$$v = \frac{(\rho_m - \rho_l) \times \frac{4}{3}\pi R^3 g}{6\pi\eta R}$$

$$v = \frac{2R^2}{9\eta}(\rho_m - \rho_l)g$$



(when forces are balanced)

For general expression,

$$v = \frac{2R^2}{9\eta}|\rho_m - \rho_l|g$$

Que.) Two identical rain drops falling with the terminal velocity of 1 m/s each combine to form a bigger drop. Find new terminal velocity.

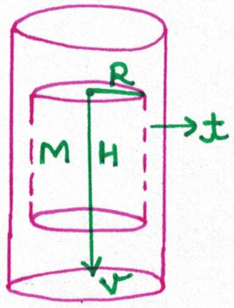
$$2 \times \cancel{\frac{4}{3}}\cancel{\pi} r^3 = \cancel{\frac{4}{3}}\cancel{\pi}(r')^3$$

$$r' = \sqrt[3]{2}\, r$$

$\therefore$ New terminal velocity $= \sqrt[3]{4}\, v$

$= \sqrt[3]{4}$ m/s

Que.) Find terminal velocity.



$$F_v = -\eta 2\pi R H \frac{v}{t}$$

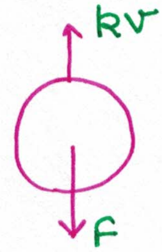
(For termination)

$$Mg = F_v$$

$$\boxed{v = \frac{Mgt}{2\pi R H\eta}}$$

NATURE OF v-t GRAPH WHILE ATTAINING TERMINAL VELOCITY

$$F - kv = M\frac{dv}{dt}$$



$$\int_0^v \frac{M\,dv}{F-kv} = \int_0^t dt$$

$$\frac{M}{-k}\Big[\ln(F-kv)\Big]_0^v = t$$

$$\frac{F-kv}{F} = e^{-\frac{kt}{M}}$$

$$v = \frac{F}{k}\left[1 - e^{-\frac{kt}{M}}\right]$$